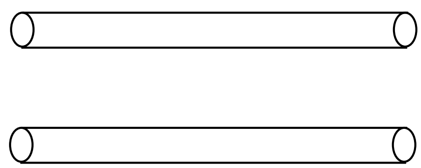
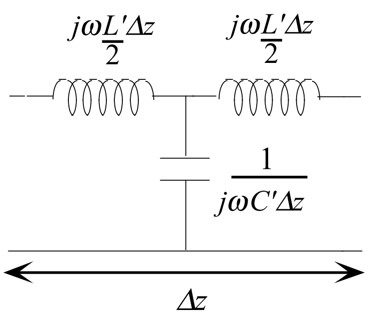
Distributed Circuits and Transmission Lines

In circuit analysis, wire are typically considered to have no electrical properties, except as to allow current to flow between components. Hence, they have no effect on the operation of the circuit. In real life, however, wires add both capacitance and inductance to the circuit. A particularly simple case to consider is when wires come in pairs of parallel conductors, such as shown below: As a pair, these wires are considered to be a transmission line, to which components such as voltage sources and loads can be attached to form a complete circuit.



It is known from electromagnetics that each short length Δz of a transmission line has series inductance and shunt capacitance, which can be represented as the T-section equivalent circuit (or, unit cell) shown below:

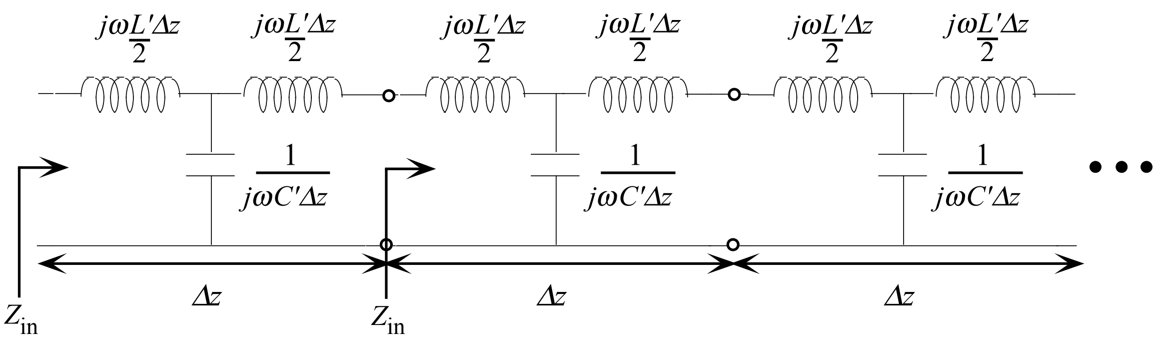


Here, L' and C' are the inductance per meter and capacitance per meter of the transmission line, respectively, and Δz is considered to be a "small enough" distance, which we'll define later. In general, C' increases as the conductors/wires are brought closer together, and L' increases, but the product is always related by:

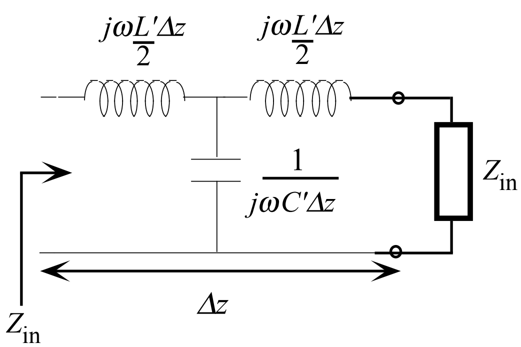
$$\frac{1}{\sqrt{L'C'}} = c \quad ,$$

where c is the speed of light in the host medium

To understand how these distributed inductances and capacitances can affect the operation of a circuit, consider an infinitely long (to the right) transmission line, modeled as an infinite chain of unit cells below.



Here, the first three cells are shown. To find the input impedance Z_{in} of the infinite line, we first note that since the line is infinite, it will have the same impedance if one cell is removed. Hence, we can remove all the cells after the first cell and replace them with a single lumped impedance of value Z_{in}, as shown in the figure below.



We can solve for the input impedance by recognizing that Z_{in} is simply the inductive impedance of the left inductor, in series with the impedance of the capacitor in parallel with the right-inductor in series with the Z_{in} of the rest of the infinite line:

$$Z_{in} = j\omega \frac{L'}{2}\Delta z + \frac{1}{j\omega C'\Delta z} \| \left[j\omega \frac{L'}{2}\Delta z + Z_{in} \right]$$

Solving for Z_{in}, we find, after some manipulation

$$Z_{in} = \sqrt{L'/C' + \left[j\omega \frac{L'}{2} \Delta z \right]^2}$$

If we make Δz small so that $\Delta z << \frac{1}{\omega\sqrt{L'C'}}$, we find

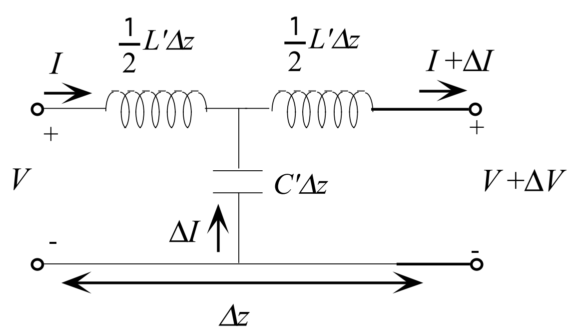
$$Z_{in} = \sqrt{L'/C'}$$

This impedance is typically called the characteristic impedance of the transmisson line, indicated by the symbol Z_o. Hence,

$$Z_o = \sqrt{L'/C'} \qquad \text{Characteristic Impedance} \qquad [1]$$

This is a remarkable result, since one would expect the input impedance of a circuit that had only inductors and capacitors to be reactive (imaginary), but clearly the fact that the circuit has an infinite number of components has caused something interesting to happen – the input impedance is resistive!

Next, to see how the voltages and currents vary as a functions of position along a transmission line, consider the T-circuit for a small section of a transmission line shown below.



Since the length of the section Δz is small, the values of the inductance and the capacitance are small, so the inductors are nearly short circuits, and the capacitor is nearly an open circ circuit.

Applying Kirchhoff's voltage law around the outer perimeter of the circuit. A clockwise KVL path around the circuit yields

$$-V+\frac{1}{2}L'\Delta z\frac{\partial I}{\partial t}+\frac{1}{2}L'\Delta z\frac{\partial(I+\Delta I)}{\partial t}+V+\Delta V=0$$

where V and I are the voltage and current at the left-hand terminals, respectively, and $V+\Delta V$ and $I+\Delta I$ are the voltage and current at the right-hand terminals, respectively. As $\Delta z \rightarrow 0$, $I+\Delta I \rightarrow I$, so

$$L'\Delta z\frac{\partial I}{\partial t}+\Delta V=0$$

Dividing both sides by Δz, yields:

$$\frac{\Delta V}{\Delta z}=-L'\frac{\partial I}{\partial t}$$

and taking the limit as $\Delta z \rightarrow 0$, this becomes

$$\frac{\partial V}{\partial z}=-L'\frac{\partial I}{\partial t} \qquad [2]$$

Next, the voltage across the shunt capacitance $C'\Delta z$ approaches V when Δz is small, so we can express the current ΔI flowing through these elements as

$$\Delta I=-C'\Delta z\frac{\partial V}{\partial t}$$

Dividing both sides of this expression by Δz, this becomes:

$$\frac{\Delta I}{\Delta z}=-C'\frac{\partial V}{\partial t}$$

which the limit as $\Delta z \rightarrow 0$ becomes

$$\frac{\partial I}{\partial z} = -C' \frac{\partial V}{\partial t} \qquad [3]$$

Taken as a pair, equations [2] and [3] describe the relationship of the current and voltage on a transmission line as a function of both time and position.

$$\frac{\partial V}{\partial z} = -L' \frac{\partial I}{\partial t} \qquad [4]$$

$$\frac{\partial I}{\partial z} = -C' \frac{\partial V}{\partial t} \quad . \qquad [5]$$

These are simple differential equations, but they are coupled, since they both contain V and I. To obtain an equation that contains only V, let us first differentiate equation [4] with respect to z, obtaining

$$\frac{\partial^2 V}{\partial z^2} = -L' \frac{\partial^2 I}{\partial z \partial t} \qquad [6]$$

Here we have assumed that I is a "well behaved" function, so the order of differentiation with respect to z and t can be interchanged. Next, if we differentiate equation [5] with respect to t, we have

$$\frac{\partial^2 I}{\partial t \partial z} = -C' \frac{\partial^2 V}{\partial t^2} \quad . \qquad [7]$$

Substituting equation [7] into equation [6], we obtain a differential equation in terms of only V,

$$\frac{\partial^2 V}{\partial z^2} = L'C' \frac{\partial^2 V}{\partial t^2} \quad . \qquad [8]$$

We can derive a similar equation for the current I by a similar sequence of steps,

$$\frac{\partial^2 I}{\partial z^2} = L'C' \frac{\partial^2 I}{\partial t^2} \quad . \qquad [9]$$

Equations [8] and [9] are called ***one-dimensional wave equations***.

Propagating Voltage Waves

To understand the nature of the voltages that can exist on lossless transmission lines, let us start by stating the general solution of the voltage wave equation (equation [8]):

$$V(t,z) = V^{+}(t-z/u) + V^{-}(t+z/u) \quad , \qquad [10]$$

Where

$$u = \frac{1}{\sqrt{L'C'}}$$

Here, $V^{+}(t)$ and $V^{-}(t)$ are arbitrary functions of a single variable and are called ***waveform functions***. To verify that equation [10] satisfies equation [8], we note that the second derivatives of V with respect to t and z are

$$\frac{\partial^2 V}{\partial t^2} = V^{+\,\prime\prime}(t - z/u) + V^{-\,\prime\prime}(t + z/u) \qquad [12]$$

and

$$\frac{\partial^2 V}{\partial z^2} = \frac{1}{u^2}\ V^{+\,\prime\prime}(t - z/u) + \frac{1}{u^2}\ V^{-\,\prime\prime}(t + z/u) \ , [13]$$

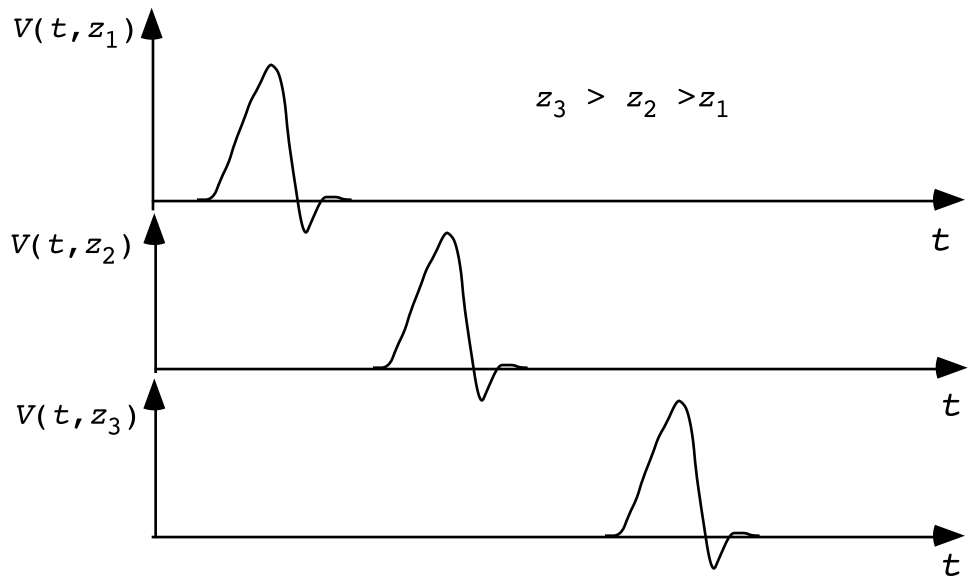
where $V^{+\,\prime\prime}$ and $V^{-\,\prime\prime}$ are the second derivatives of V^{+} and V^{-}, respectively. Substituting equations [12] and [13] into equation [8], we find that equation [10] is indeed the general solution of the wave equation for all waveform functions $V^{+}(t)$ and $V^{-}(t)$, provided that the parameter u is given by equation [11].

The voltage expression given by equation 11.22 consists of waves that travel along the transmission line. To show this, let us for the moment assume that $V^{-}(t) = 0$. For this case, the voltage expression becomes

$$V(t,z) = V^{+}(t-z/u) \quad .$$

The figure below shows $V(t,z)$ as a function of time t for three values of z when $V^{+}(t)$ is a "pulse-like" function. As can be seen, the same waveform is observed at each position, with a time-delay that increases linearly with z. Since the waveform shape is the same for all values of z, we call this ***distortionless*** (or ***dispersionless***) ***propagation***. This is a characteristic of all lossless transmission lines. To calculate the propagation velocity, let us observe how fast the value of z must change in order for an observer to "ride" on the same point of the pulse as it moves. This occurs when the argument of V^+ remains constant as time progresses, $t - z/u$ = constant . Differentiating both sides of this expression with respect to t, we obtain

$$\frac{dz}{dt} = \frac{1}{\sqrt{LC}} = u \; .$$



Thus, we can conclude that the waveform $V^+(t\text{-}z/u)$ travels (i.e., propagates) towards increasing values of z at a rate of

$$u = \frac{1}{\sqrt{LC}} \quad [\text{m/s}] \,, \qquad [14]$$

where u is called the ***velocity of propagation***. Waves propagating towards increasing values of z are called ***forward-propagating waves***.

Returning to the general voltage expression given by equation [10], let us now consider the case where $V^+ = 0$ and $V^- \neq 0$. For this case, we have

$$V(t,z) = V^-(t+z/u) \quad .$$

To "ride" on the same point of this waveform, we must maintain

$$t + z/u = \text{constant} \quad .$$

Differentiating both sides of this expression with respect to t, we obtain

$$\frac{dz}{dt} = -u = -\frac{1}{\sqrt{LC}} \quad ,$$

which means that the term $V^-(t+z/u)$ represents a wave traveling in the $-z$ direction at the rate $|u| = \frac{1}{\sqrt{LC}}$. We will call waves propagating in this direction ***backward-propagating waves***.

Propagating Current Waves

Associated with each traveling-wave voltage is a traveling-wave current. To show this, we first remember from equation [9] that the current $I(t,z)$ satisfies exactly the same one-dimensional wave equation that $V(t,z)$ does,

$$\frac{\partial^2 I}{\partial z^2} = L'C'\frac{\partial^2 I}{\partial t^2} \quad . \qquad [15]$$

Hence, just as with $V(t,z)$, solutions for $I(t,z)$ are always of the form

$$I(t,z) = I^+(t-z/u) + I^-(t+z/u) \quad , \qquad [16]$$

where u is given by equation [14]. Although it may appear from equation [15] that the waveform functions $I^+(t)$ and $I^-(t)$ are arbitrary, they have the same shapes as the forward-propagating and backward-propagating voltage waveform functions, $V^+(t)$ and $V^-(t)$, respectively. To show this, substitute equations [10] and [16] into equation [4], obtaining

$$-\frac{1}{u}\, V^+(t-z/u) + \frac{1}{u}\, V^-(t+z/u)$$

$= -L\, I^{+}(t\text{-}z/u)\ \ - L\, I^{-}(t\text{+}z/u)$.

Both sides of this equation will be equal for all values of t and z only when

$$\frac{V^{+}(t)}{I^{+}(t)} = Z_o$$

and

$$\frac{V^{-}(t)}{I^{-}(t)} = -\, Z_o \quad ,$$

where

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

is the characteristic impedance of the line. Hence, the voltage and current on the line can be finally written as

$$V(t,z) = V^{+}(t - z/u) + V^{-}(t + z/u)$$

$$I(t,z) = \frac{1}{Z_0} V^{+}(t - z/u) - \frac{1}{Z_0} V^{-}(t + z/u)$$

Here, we see that the forward voltage and current waves propagate as a unit, with the same velocity and a ratio equal to the characteristic impedance of line. Similarly,t he bacward voltage and current waves also propagate as a unit, with the same velocity, but with a ratio equal to the negative of the characteristic impedance of line.

The choice of the name for parameter Z_o *, **characteristic impedance,*** is a logical one, since it is measured in Ohms and is the ratio of a voltage and a current. However, this impedance, although real-valued, is *not* like lumped resistors, which dissipate electrical energy. Rather, the characteristic impedance of a transmission line is an indication of its ability to transport energy via the propagation of voltage and current waves.

Launching Waves on Transmission Lines

A wave can be launched on a transmission line simply by attaching a voltage across its terminals. The Figure a depicts such a situation. Here, an independent voltage generator $V_g(t)$ and a resistor R_g are connected to the end of an infinite, lossless transmission line. The waveform shape of $V_g(t)$ is shown in the Figure, which has a peak amplitude of A. Since the line is infinitely long, the total voltage and current on the line consist only of forward-propagating waves,

$$V(t,z) = V^{+}(t\text{-}z/u) \tag{11.46}$$

and

$$I(t,z) = I^{+}(t - z/u) = \frac{1}{Z_0} V^{+}(t - z/u) \tag{11.47}$$

Since only a forward-propagating wave exists on the line, the impedance Z_{in} looking into the line at z =0 is the same for all time t :

$$Z_{in} = \frac{V(t,0)}{I(t,0)} = \frac{V^{+}(t - z/u)}{\frac{1}{Z_0} V^{+}(t - z/u)} = Z_0 \,.$$

Because of this, the input circuit can be redrawn as shown in Figure 11-11b, where the infinite transmission line has been replaced by a resistor of value Z_o . Using the voltage divider relation, we obtain

$$V(t,0) = \frac{Z_0}{R_g + Z_0} V_g(t)\,, \tag{11.48}$$

which is the amplitude of the transmission line voltage at z = 0. Substituting this result into equations 11.46 and 11.47, we obtain the following voltage and current waves:

$$V(t,0) = \frac{Z_0}{R_g + Z_0} V_g(t) \tag{11.49}$$

and

$$I(t,0) = \frac{1}{R_g + Z_0} V_g(t)\,. \tag{11.50}$$

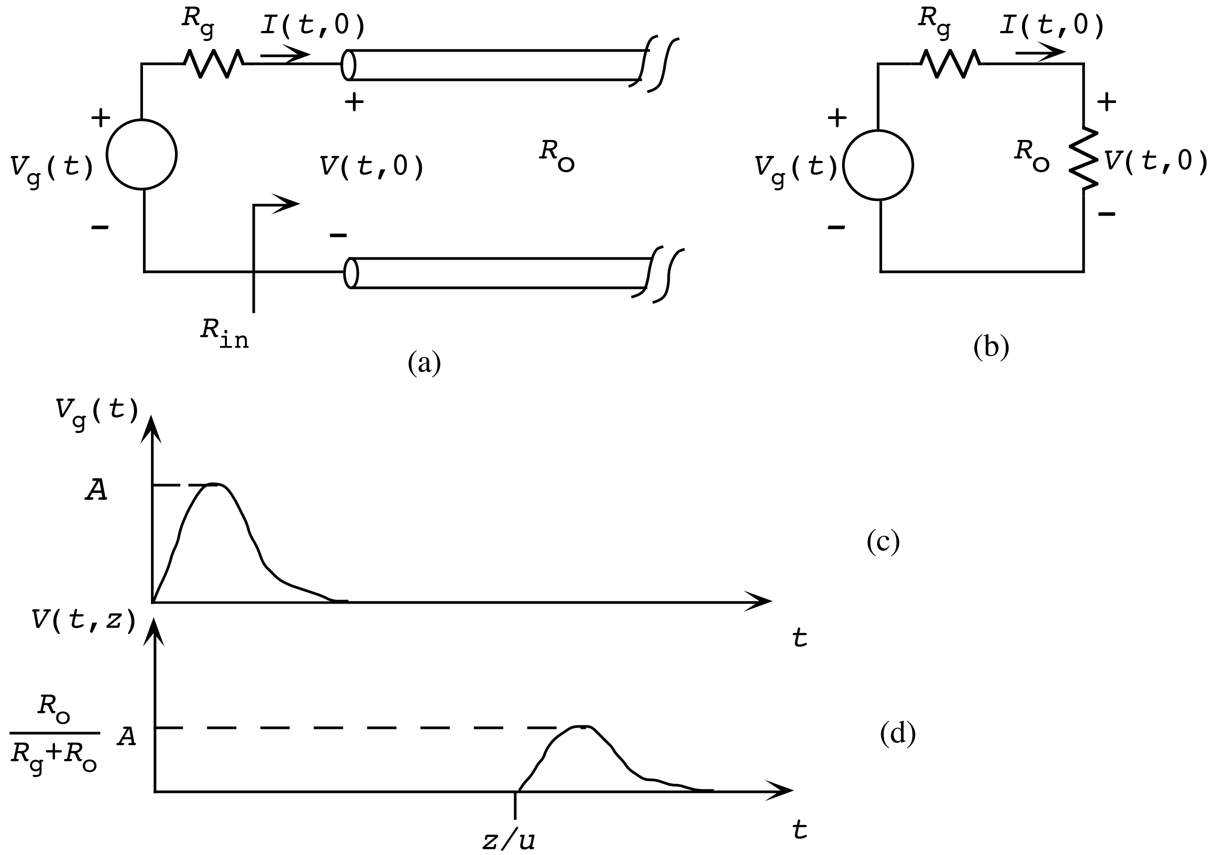


Figure 11-11: Launching waves on a transmission line: a) A voltage generator connected to an infinite transmission line, b) the equivalent circuit as seen by the generator circuit, c) the voltage generator waveform, d) the voltage waveform observed a distance z along the transmission line.

Figure 11-11d shows that $V(t,z)$ is simply a delayed and attenuated version of the generator waveform $V_g(t)$. Since we assumed that the transmission line is infinitely long, the waves launched by the generator will propagate forever without encountering any discontinuities. Because of this, no backward-propagating waves will appear. In the next section, we will investigate what happens when transmission lines are terminated with lumped resistors.

11-3.4 Reflections From Resistive Terminations

Figure 11-12a shows a section of lossless transmission line with characteristic resistance Z_o, terminated with a load resistance of value R_L at $z = z'$.

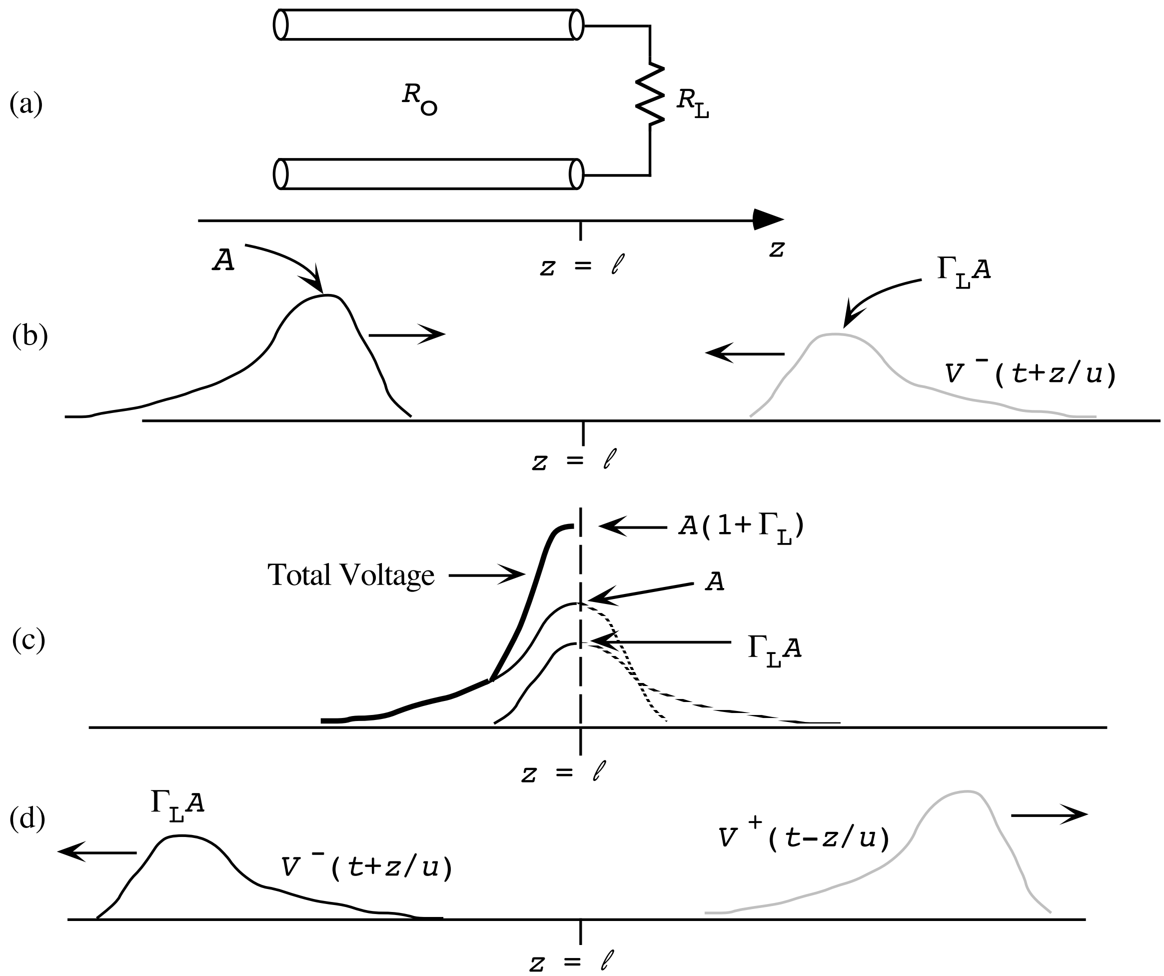


Figure 11-12: The process of reflection at a resistive load: a) a transmission line terminated by a resistor, b)- d) line voltage along the line before, during, and after the incident pulse reaches the resistor, respectively.

We will assume that a source far off to the left of the figure has launched forward-propagating (or incident) voltage and current waveforms that are described by

$$V_{\text{inc}}(t,z) = V^+(t-z/u) \qquad 11.51$$

And

$$I_{inc}(t,z) = I^+(t-z/u) = \frac{1}{Z_0}V^+(t-z/u), \qquad 11.52$$

where $V^+(t\text{-}z/u)$ has a peak amplitude of A. If we assume that the waveform $V^+(\tau)$ is zero for $\tau < 0$, the leading edges of the incident waves will not reach the load until $t = \ell/u$. Thus, $V^+(t\text{-}z/u)$ and $I^+(t\text{-}z/u)$ are the only waves on the line for $t < \ell/u$.

When the incident waves reach the load, backward-propagating waves will be initiated at the load if $R_L \neq Z_o$. To see why, let us suppose that only the forward-propagating waves are present on the line for all values of t. If this were the case, the load voltage $V_L(t)$ and current $I_L(t)$ would simply be the incident waves, evaluated at $z = \ell$,

$$V_L(t) = V(t,\ell) = V^+(t - \ell/u)$$

and

$$I_L(t) = I(t,\ell) = \frac{1}{Z_0} V^+(t - \ell / u) .$$

However, at the load, the ratio of the voltage and current must equal the load resistance R_L

$$\frac{V_L(t)}{I_L(t)} = R_L \quad . \qquad 11.53$$

Substituting the expressions for $V_L(t)$ and $I_L(t)$ into this equation, we find that the load resistance must be

$$R_L = Z_o \quad .$$

A load that is equal to the characteristic resistance produces no reflections and is called a ***matched load***. When $R_L \neq R_o$, equation 11.53 is not satisfied, which means the incident waves alone cannot satisfy the conditions of both the transmission line and the load.

To model the case where $R_L \neq Z_o$, let us again assume that the same forward-propagating waves $V^+(t\text{-}z/u)$ and $I^+(t\text{-}z/u)$ are incident from the left in Figure 11-12a, but this time let us also speculate that reflected, backward-propagating waves are also present. Hence, the total voltage and current on the line are given by

$$V(t,z) = V^+(t\text{-}z/u) + V^-(t\text{+}z/u) \qquad 11.54$$

and

$$I(t,z) = I^+(t - z/u) + I^-(t + z/u) = \frac{1}{Z_0} V^+(t - z/u) - \frac{1}{V_0} V^-(t + z/u) \quad , \qquad 11.55$$

where $V^-(t)$ is a yet-to-be-determined reflected waveform. Also, note that the negative polarity of the reflected current $I^-(t+z/u)$ occurs because this wave is backward-propagating (see equation 11.32). Evaluating these expressions at $z=\ell$, the voltage $V_L(t)$ and current $I_L(t)$ at the load are

$$V_L(t,z') = V^+(t,\ell) + V^-(t,\ell)$$

and

$$I_L(t,z') = \frac{1}{Z_0}V^+(t,\ell) - \frac{1}{Z_0}V^-(t,\ell).$$

From these expressions, the ratio of the load voltage and load current is

$$\frac{V_L(t)}{I_l(t)} = \frac{V^+(t,\ell)+V^-(t,\ell)}{\frac{1}{Z_0}V^+(t,\ell)-\frac{1}{Z_0}V^-(t,\ell)}.$$

Setting this expression equal to the load resistance R_L and solving for $V^-(t)$, we obtain

$$V^-(t,\ell) = \Gamma_L V^+(t,\ell) , \qquad 11.56$$

where Γ_L is the ***reflection coefficient***, defined by

$$\Gamma_L \equiv \left.\frac{V^-(t)}{V^+(t)}\right|_{\text{at the load}} = \frac{R_L - Z_0}{R_L + Z_0}. \qquad 11.57$$

Substituting equation 11.56 into equations 11.54 and 11.55, we obtain the complete expressions for the voltage waves on a terminated line,

$$V(t,z) = V^+(t-z/u) + \Gamma_L V^+(t+z/u) \qquad 11.58$$

and

$$I(t) = \frac{1}{Z_0}\left(V^+(t-z/u) - \Gamma_L V^+(t+z/u)\right). \qquad 11.59$$

These voltage and current waves satisfy the requirements of both the transmission line and the load resistance.

From equation 11.58, we see that the reflected voltage waveform has the same shape as the incident waveform, with an amplitude that is governed by the reflection coefficient Γ_L. For passive load resistances ($R_L \geq 0$), Γ_L has a magnitude that is always less than or equal to unity;

$$-1 \leq \Gamma_L \leq 1 \quad . \tag{11.60}$$

Notice that $\Gamma_L = 0$ when $R_L = Z_o$, which means that no reflection is generated by a matched load. For this case, all the power in the incident voltage and current waves is dissipated by the load.

Figures 11-12b-d shows the incident, reflected, and total voltages on the terminated transmission line for three values of t. When $t_1 < \ell/u$ (Figure 11-12b), the leading edge of the incident wave has yet to reach the load, so only the incident wave appears on the line. Even so, it is convenient to show the yet-to-appear reflected wave as a dotted curve to the right of the load position ($z=\ell$) that propagates towards the left. The peak amplitude of this reflected wave is $\Gamma_L V_p$, where V_p is the peak amplitude of the incident wave. Figure 11-12c and 11-12d show the voltages at two instants in time after the incident waveform has reached the load. In these plots, the incident waveform is drawn as a dotted line in the region $z>L$ to remind us that this region of the graph does not represent actual points on the transmission line. In Figure 11-12c, the incident and reflected waveforms appear simultaneously across the load, since the reflected wave is generated at the load the instant the incident wave appears. Figure 11-12d shows that once the incident wave has encountered the load, only the reflected wave is left on the line (assuming that there is no mismatch at the generator).

11-3.5 Step Response of Transmission Lines

We are now ready to discuss the full transient response of transmission lines that are terminated at both ends. To introduce this topic, consider the set-up shown the figure below. Here, a transmission line with characteristic resistance $Z_o = 50$ [Ω] and length $\ell = 3$ [m] is connected to a load resistor $R_L = 100$ [Ω]. The source consists of a 12 [V] battery, a resistor $R_g = 10$ [Ω], and a switch that closes at $t = 0$. Also, the velocity of propagation is $u = 3\text{x}10^8$ [m/s], so the one-way propagation delay from end to end is 10 [ns].

When the switch closes at $t = 0$, a step waveform is launched towards the load with an amplitude V_1 given by equation 11.49,

$$V_1 = \frac{50}{50+10}\ 12 = 10\ [\mathrm{V}]\ .$$

For 0<t<10 [ns], this is the only voltage wave on the line. Figure 11-13b shows the line voltages at t = 7 [ns].

At t=10 [ns], the leading edge of the incident waveform reaches the load, where a reflected wave is produced. The reflection coefficient at the load end is

$$\Gamma_L = \frac{100\text{-}50}{100+50} = \frac{1}{3}\ ,$$

so the first reflected wave has amplitude

$$V_2 = V_1\,\Gamma_L = 10 \text{ x } \frac{1}{3} = 3.3333 \text{ V}\ .$$

The following figure 11-13c shows the line voltages at t=17 [ns].

The first reflection from the load reaches the generator terminals at t =20 [ns]. Since the generator resistance is not matched to the transmission line, a reflected wave will be produced that propagates towards the load. The amplitude of this reflected wave is not affected by the battery, since, according to the superposition principle, the battery voltage has already been accounted for in the first forward-propagating wave (launched at t=0). The reflection coefficient Γ_g at the generator end is

$$\Gamma_g = \frac{10\text{-}50}{10+50} = -\frac{2}{3}\ ,$$

so the reflection of V_2 off the generator resistance is

$$V_3 = \Gamma_g\,V_2 = -\frac{2}{3} \text{ x } 3.3333 = -2.222 \text{ V}\ .$$

Figure 11-13d shows the voltages on the line at t=27 [ns].

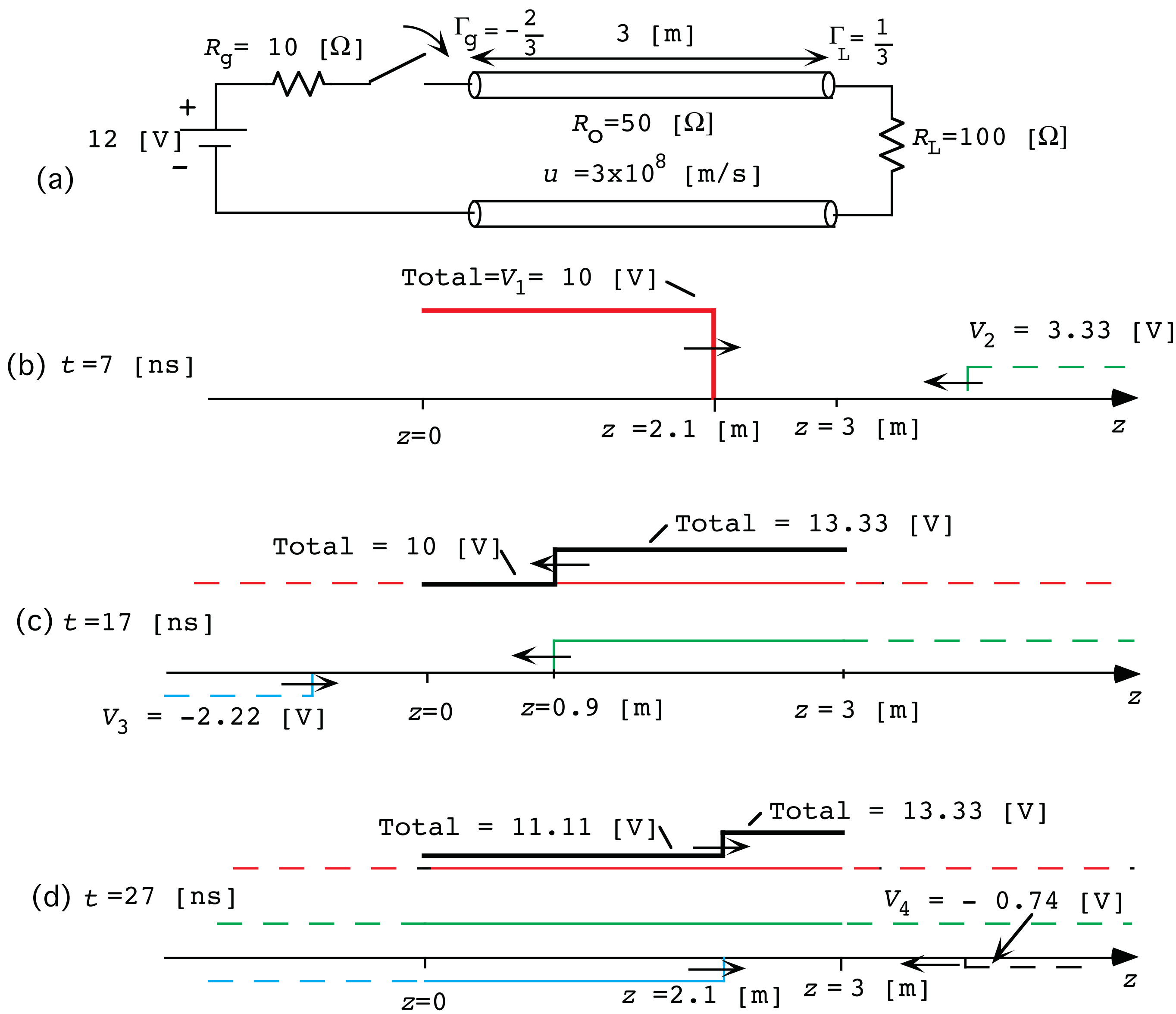


Figure 11-13: Transient response of a transmission line, switched at t = 0: a) the circuit, b)-d) voltage waveforms on the line at three points in time. The arrows show the propagation directions of the leading edges of the waveforms.

By now, the method for determining the subsequent reflections on the line should be obvious. To determine Nth reflection at either the generator or load, all that must be known is the amplitude of the approaching (N-1)th wave and the reflection coefficient. In this way, the total voltages on the line can be considered as an infinite sum of reflections. Since the reflection coefficients of passive loads have magnitudes less than or equal to the previous one, the higher order reflections eventually have negligible amplitudes. As a result, the step response of a transmission line approaches a constant value along the entire line as $t \to \infty$.

Figures 11-13b through 11-13d show "snapshots" of the voltages on the line at various points in time. Plots like these give a global picture of how the waves reflect and re-reflect off the terminations. Another useful way to determine the step response of a transmission line is by using a ***bounce diagram***, such as the one shown in the Figure below.

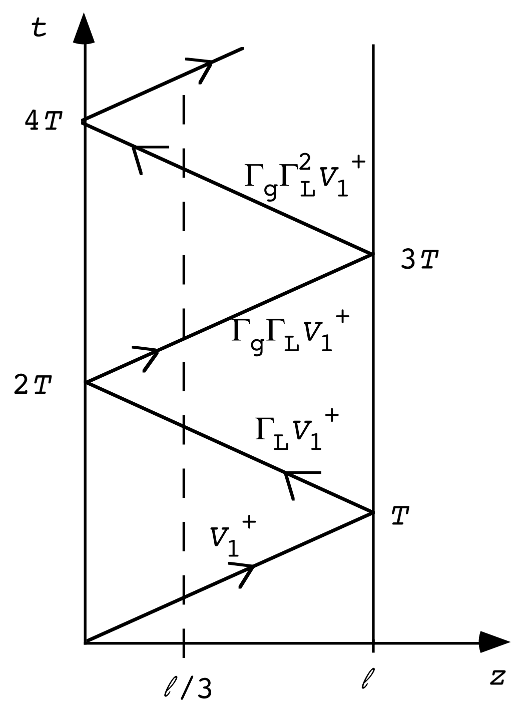
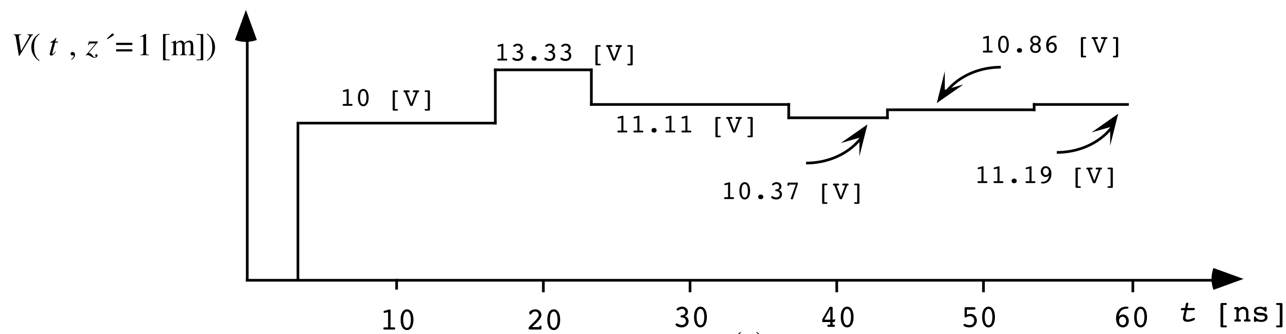


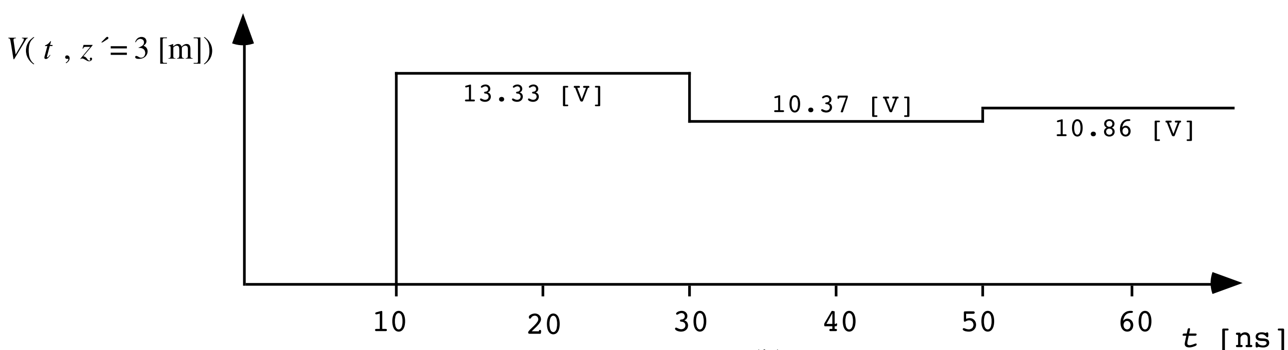
Figure 11-14: A transmission line bounce diagram.

In a bounce diagram, the progression of the leading edges of the incident and reflected voltage waves are displayed as functions of both time and position. In Figure 11-14, the line marked "V_1^+" indicates the progress of the leading edge of the wave launched by the generator as it propagates towards the load. This line starts at (t=0,z=0) and ends at (t=T,z=ℓ), where $T = \ell/u$ is the one way transit time. The line marked $\Gamma_L V_1^+$ represents the first reflection off the load. This line starts at $t = T$ and has a negative slope, since it represents a backward-propagating wave. In like manner, each of the subsequent reflections are represented by lines that have alternating positive and negative slopes and begin at progressively later times.

To obtain the voltage waveform $V(t,z')$ at a point $z = z'$ on a transmission line, we first draw a vertical line at $z = z'$ on the bounce diagram. Next, starting at t=0 and $z = z'$, we progress vertically on this line, noting the times t_n where this line intersects the lines representing each wave. At each value of t_n, the waveform $V(t,z')$ will exhibit a step discontinuity equal to the value of the newest wave arriving at that point. Figures 11-15a&b show $V(t,z')$ at z'=1 [m] and z' =3 [m], respectively, for the transmission line network shown in Figure 11-15a when the transmission line has length ℓ= 3 [m].



(a)



(b)

Figure 11-15: Voltage waveforms for the circuit in Figure 11-15a: a) z' = 1 [m], b) z' = 3 [m]

In particular, the waveform at z' = 3.0 [m] has fewer jumps in this time interval than the waveform at z' = 1.5 [m]. This is because an observer at the load (z' = 3.0 [m]) "sees" the leading edges of the incident wave and its reflection simultaneously, whereas an observer in the center of the transmission line sees them at different times.